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Letter

A Minimum Value for the Density of Random Close-Packing of Equal Spheres

The density of random close-packing of equal spheres has been the subject of empirical study for a considerable period, and a value of about 0.61 is generally accepted. Scott [1] has distinguished between "loose random packing" with a density of 0.60 and "dense random packing" with a density of 0.64. A computer model of random packing yielded a density of 0.609 [2]. The theoretical upper limit for the density of packing corresponds to that of four touching spheres at the vertices of a regular tetrahedron, and has the value 0.780 [3], which is somewhat greater than the densest regular packing. It is known that tetrahedra cannot be stacked together to fill space, so that this value is unlikely to be realised in practice.

No theoretical value for the density of a random packing appears to exist, but it will be shown that a minimum value can be calculated by considering the similarity between the random packing of spheres and the shape of grains in a single-phase polycrystalline material. Various authors have compressed random packings of spheres until all porosity was removed, and have shown that the resulting cell shapes are closely similar to those of grains in a polycrystalline material [4]. The reason for the similarity is that in both cases space is divided in the most economical fashion. No unique cell shape exists, but the average cell can be shown to have 5.10 edges per face, 13.4 faces, and 22.8 vertices [5]. The derivation of these values assumes that the edges of the cells are straight.

The geometry of the cells may thus be taken as defining the positions of the centres of the spheres in random packing, and the density of such a packing will then be given by the ratio of the volume of the sphere inscribed in the average cell to the volume of the average cell. The metric properties of the average cell can be calculated [6] by applying the standard formulae which apply to the regular polyhedra, yielding the value 9.871^3 for the volume of the average cell, and $1\sqrt{3/2}$ for the radius of the inscribed sphere, where 1 is the length of the (equal) cell edges. It follows that the density of the packing is 0.780, in agreement with the value for a single tetrahedral arrangement, as might be expected from the fact that the cells meet four to a point. The arrangement described is thus an extended one consisting of packed tetrahedra of spheres, and the derivation shows at once that the co-ordination number of the average sphere is 13.4. It is known that the co-ordination of a real packing cannot exceed 12, so that the packing considered above must dilate further in practice. A sufficient dilation, keeping the spheres in the same relative positions, will be obtained by increasing the sphere-to-sphere distance from twice the inradius of the average cell to the diameter of the sphere of volume equal to the cell. This dilation is 1/0.780 in volume terms, and gives a minimum density for the random packing. The value of this minimum density is obviously $(0.780)^2$, which is 0.608, a value in excellent agreement with experiment.

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In view of the reproducible nature of this packing, the description "random" seems hardly appropriate, and it is suggested that "irregular close-packing" would be a better name.

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Book Reviews

Diffusion Data, Vol. 1 No 3 (December 1967)

Edited by R. H. Wohlbier

Pp viii + 146 (Diffusion Information Center, Cleveland, Ohio, USA) $38 per annum

All active research workers are conscious of the difficulty of maintaining familiarity with the recent literature of their subject. Flourishing abstract journals testify to this difficulty but even skimming through the index of such journals has become a formidable task. Critical reviews also have their place but in general these are not useful for recent work. Clearly there is a place for an intermediate type of journal neither as comprehensive as the abstracting journals nor as carefully digested as review articles.

The periodical under review is a good example of such a journal. It claims to include "all data on mass transport in and through inorganic solids and their melts . . . within a few weeks after their publication". This is an ambitious claim but spot checks substantiate it. For this reason the journal is clearly one to which anyone interested in the properties of materials must have access. Because of the wide range of materials covered, the journal will certainly provide opportunities for cross fertilisation between disciples. From this point of view it is disappointing that solid inert gases and organic solids have been excluded. Diffusion in these substances is of considerable interest. In particular the study of mass transport in organic solids promises to become one of the more important branches of diffusion in the near future.

There are other minor criticisms which must be made. In a journal of this kind, success depends on the case with which information can be located. With the present arangement five separate steps are necessary to obtain full information about a paper. These are (i) looking up the system of interest in the table of contents; (ii) finding the page where the first author's name and the abstract can be found; (iii) looking up the journal reference supplied in the abstract; (iv) looking up the author's name in the index to obtain an address reference; (v) looking up the address reference. This number of steps is excessive. Putting the authors' names, the journal reference and the address with the abstract would seem preferable.

It must be admitted, however, that the index of laboratories studying diffusion, classified by country, is very useful, particularly in planning visits. This usefulness is much reduced as there is no simple way of locating an author from the laboratory address list. It is pleasing to me personally to find the journal sufficiently up to date to list Scotland and England separately.

Another error is that the equation defining the isotope effect in the introduction (repeated in each issue which seems unnecessary) is incorrect. The square root should include only the isotopic masses. The form in which this equation is given is clumsy. The original form used by Schoen,

$$E = \left(\frac{D_\beta}{D_\alpha} - 1\right) \Big/ \left[\left(\frac{m_\alpha}{m_\beta}\right)^{\frac{1}{2}} - 1\right], \quad m_\alpha > m_\beta$$

is much more convenient and preferable on the grounds of priority.

F. J. P. CLARKE